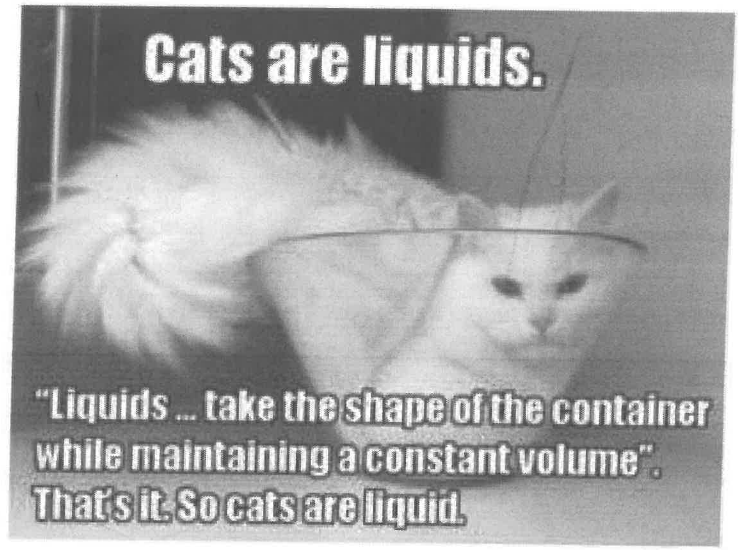
Chapter 1: MEASUREMENT

Allwdi

Conversions:

1 cm	=	10 mm
1 inch	=	2.54 cm
1 foot	=	0.3048 m
1 foot	=	12 in
1 yard	=	3 ft
1 metre	=	100 cm
1 metre	=	3.280839895 ft
1 furlong	=	660 ft
1 km	=	1000 m
1 km	=	0.62137119 mi
1 mile	=	5280 ft
1 mile	=	1.609344 km



$$A_{cylinder} = 2\pi r^2 + 2\pi rh$$

$$A_{rect.prism} = 2(lw + wh + lh)$$

$$V_{cylinder} = \pi r^2 h$$

$$V_{rect.prism} = lwh$$

$$A_{sphere} = 4\pi r^2$$

$$A_{hemisphere} = 3\pi r^2$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$V_{hemisphere} = \frac{2}{3}\pi r^3$$

$$A_{right.pyramid} = \frac{1}{2}s(base.perimetre) + (base.area)$$

$$A_{cone} = \pi r^2 + \pi rs$$

$$V_{pyramid} = \frac{1}{3}lwh$$

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

SECTION 1.1: IMPERIAL MEASURES OF LENGTH p. 4

In 1976, Canada adopted the **SI units** (Système International d'Unités, or Metric) to measure length. Many construction and manufacturing industries and trades continue to use imperial units, or a mix of both units. Today, Canadians typically use a mix of metric and imperial measurements in their daily lives, and so it is important to be "fluent" in both systems. We tend to know our height (ft) and weight (lbs) and size of our houses (square feet) in **imperial units**. But we generally use **metric** for weather in degrees Celsius, buy gasoline and milk in litres, set speed limits in kilometres per hour (km/h), read road signs and maps displaying distances in kilometres, talk of rain and snowfall in millimitres and centimetres, and say how many pounds we can "bench" (lift in chest press). The United States and in many ways Britain continue to generally use the Imperial system (although metric is used in various industries).

In the Imperial system, more than 300 different units exist to measure various physical quantities.

Name some imperial units of measure that you know: inches

feet miles yard

teaspoon cup gallon

ounce tablespoon cup

This system is NOT a decimal system. This system is based on **referents. A <u>referent</u> is an item an individual uses as a measurement unit for estimating**. The Imperial measurement system started in Ancient Roman Times. The current Emperor's foot length would have been the standard unit for measuring length (distances). This resulted in units that were different in different regions. In 1824, the units were standardized and became the Imperial System of Measurement. The length of a foot was standardized to equal 12 inches. Many units in the imperial system are based on the measurements of the human body.

A. **Referents for Measurement Systems**

In measurement, a **referent is a concrete object that approximates a measurement.** Some common referents are given in the table below for the Imperial system although a referent can be almost anything that is useful.

unit	abbreviation	symbol	referent	Relationship between units
inch	in	"	Length of thumb from tip to knuckle or width of thumbnail 	
foot	ft	'	Foot length  or Length from wrist to elbow	1 ft = 12 in
yard	yd		Arm span or Width of doorway or Normal walking stride 	1 yd = 3 ft 1 yd = 36 in
mile	mi		Distance walked in 20 minutes or 2000 steps or 15-20 city blocks or Distance average person can jog in 10 minutes 	1 mi = 1760 yd 1 mi = 5280 ft

We can use referents if we don't have a measuring tool and simply want a quick estimate. If we wanted to estimate the length of a room in feet, we could walk heel to toe across the room, counting how many steps we took.

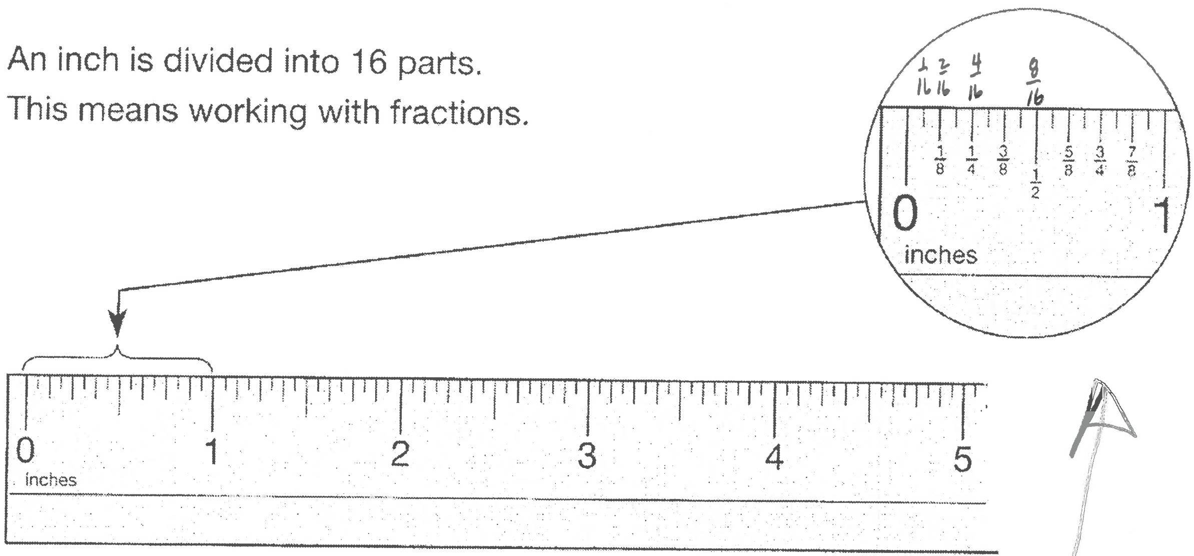
Estimate the length of your writing utensil in inches, using a referent:

__4__ (We can write __4__ inches; __4__ in; or __4__ ")

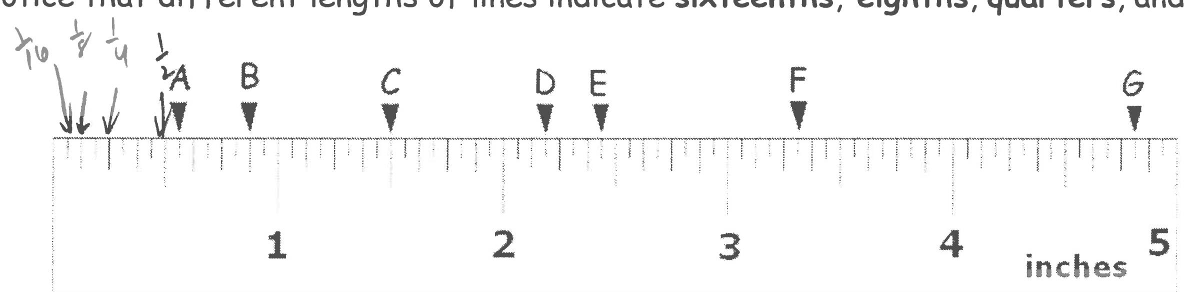
B. Imperial Measurement

Imperial lengths can be measured with rulers, yard sticks, or calipers. Many rulers marked with imperial units show one inch divided into quarters, eighths, and sixteenths. A fraction of an imperial measure of length is usually written in fraction form, **not decimal** form.

An inch is divided into 16 parts.
This means working with fractions.



Notice that different lengths of lines indicate **sixteenths**, **eighths**, **quarters**, and **halves**.



Write the measurement in inches indicated by the following arrows, to the nearest 16th of an inch. Simplify fraction if possible.

A $\frac{9}{16}$" B $\frac{14}{16}$" C $1\frac{8}{16}$" D $2\frac{3}{16}$" E $2\frac{7}{16}$" F $3\frac{5}{16}$" G $4\frac{13}{16}$"

"9 16ths of an inch"

= $\frac{7}{8}$" "7 8ths" = $1\frac{1}{2}$"

A common abbreviation for 5 feet 2 inches is 5 ft. 2 in. or 5' 2". (5.2 is the same as 5 $\frac{2}{10}$)

$5\frac{2}{12}$

Note: 5 ft. 2 in. is NOT the same as 5.2 feet. See figure 1.

1 foot = 12 inches
1 foot is divided into 12 sections (inches)

$\frac{2}{12} = \frac{1}{6} = 0.1\overline{6}$
5ft 2in = 5.16 feet

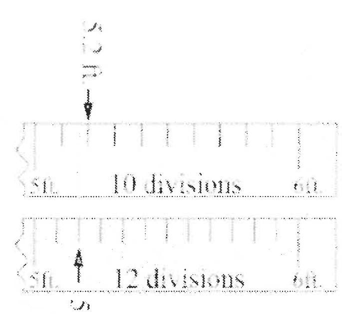


figure 1.

C. Conversion within the Imperial System

- To convert larger units to smaller units : MULTIPLY

To convert a larger unit to a smaller unit (ex ft to in),

1. First check the number of smaller units needed to make 1 larger unit.
2. Next, ~~DIVIDE~~ MULTIPLY that number by the number of larger units

When converting a larger unit to a smaller unit, the number increases (gets bigger)

Think it through logically. If 1 foot = 12 inches, to find 6 feet in inches, would we multiply or divide? Should we have a larger or smaller number of inches than feet? Try it:

$\boxed{6 \times 12 = 72 \text{ inches}}$ You would expect a larger number.

or

$6 \div 12 = 0.5$ inches

Is 6 feet = 0.5 inches? (No!)

Example 1: Convert 7 yd to i) ft ii) in

i) Since 1 yd = 3 ft, we must multiply 7 yd by 3 to get ft

$7 \times 3 = 21$

7 yds = 21 ft

ii) On our formula sheet, we are not given the number of inches in a yard. But we know the number of inches in a foot and number of feet in a yard.

Since 1 yd = 3 ft, we must multiply 7 yd by 3 to get ft (*as in i above*)

Next since 1 foot = 12 inches, we must multiply 21 ft by 12 to get inches.

21×12

Or: $7\text{ yd} \times \frac{3\text{ ft}}{1\text{ yd}} \times \frac{12\text{ in}}{1\text{ ft}} = 7 \times 3 \times 12 \text{ in.} = 252 \text{ in.}$

7 yds = 252 in

(Unit analysis see p. 8)

Example 2: Convert 5' 2" (5 ft 2 in) to inches:

1. **Convert 5' to inches.** 5x12=60 5 ft = 60 inches

2. **Add 2 inches to your total in (1.↑)** 60+2=62 5 ft 2 in = 62 inches

60 inches + 2 inches = 62 inches

- **To convert smaller units to larger units : DIVIDE**

1. **First check the number of smaller units needed to make 1 larger unit.**
2. **Next, DIVIDE that number by the number of larger units**

When converting a smaller unit to a larger unit, the number gets smaller (decreases)

Example: 12 inches = how many feet? 1

↑ smaller

Example 3: Convert 51 in to: i) feet ii) feet and inches ii) yards, feet and inches

i) **: Convert 51 in to: feet**

Since 1 ft = 12 in, we must divide 51 in by 12 to get ft

51÷12

51 in = 4.25 **ft**

ii) **Convert 51 in to: feet and inches**

Since 1 ft = 12 in, we must **divide 51 in** by 12 to get ft. 4.25 feet

- Take the **whole** number of feet, and multiply by 12 to get back to inches. 4x12=48

  4 feet x 12

- To find the **remainder**, subtract inches from the initial total. **The remainder is the inches amount.**

  51 - 48 = 3

- Write amount as feet and inches **51 in =** 4 **ft** 3 **in**

iii) **Convert 51 in to: yards, feet and inches**

It may be easier to first calculate how many inches are in a yard.

1 foot = 12 inches. 1 yard = 3 feet

Therefore 1 yard = 3 feet = 3x12 = 36 inches

How many inches are in 1 yard?
1 yd = 3 ft and 1 ft = 12 in.
3 ft/yd × 12 in./ft = 36 inches in 1 yard

Since 1 yd = 36 in, we must **divide** **51 in** by 36 to get yards. $1.41\overline{6}$

- Take the **whole** number of yards, and multiply by ~~12~~ 36 to get back to inches. 36"
  1 yd × 36 in
- To find the **remainder**, subtract inches from the initial total.

  51 - 36 = 15"   1yd 15"

- To find the number of feet in the remainder, divide by 12. 15÷12 = 1.25
- Take the **whole** number of feet, and multiply by 12 to get back to inches. 12"
  1ft × 12
- To find the **remainder**, subtract inches from the initial total. **The remainder is the inches amount.**

  15 - 12 = 3

51 inches = 1 yd 1 ft 3 in

The previous method is perhaps best for one-step conversions: You use one conversion factor (the equivalence between two measures or units) to convert from the one unit to the other.

But sometimes conversions are more complicated, or you're not sure which unit is "bigger". For these sorts of conversion, we use as many conversion factors as we need, setting up a long multiplication so the **units** we don't want cancel out. Note: this is not numbers "cancelling out", like when you're multiplying fractions. Instead, this is treating the units ("feet", "miles", "seconds", etc) as though they were numbers, and cancelling them.

How do I know which way to put the ratios? How do I know which units go on top and which go underneath? Not important. Instead, start with the given measurement, write it down complete with its units, and then put one conversion ratio after another in line, so that whichever units you don't want are eventually cancelled out. If the units cancel correctly, then the numbers will take care of themselves.

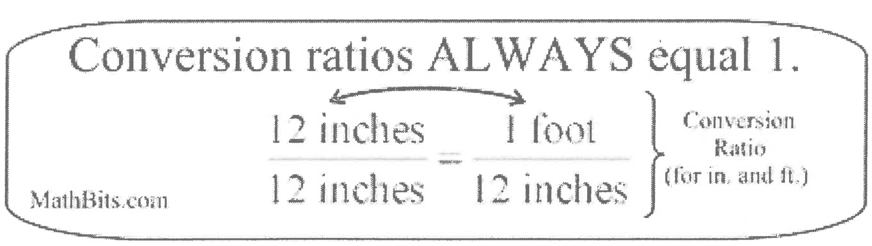
The fact that the conversion can be stated in terms of "1", and that **the conversion ratio equals "1"** no matter which value or unit is on top, is crucial to the process of cancelling units.

Setting up a unit cancellation table helps keep units straight. These tables are particularly useful when more than one unit conversion is necessary to obtain the desired unit.

Unit Analysis

Unit analysis is a method of converting or changing measures from one unit to another by multiplying the measure by a unit conversion factor in the form of a ratio (see p. 1 conversion factors). These conversion ratios specify how one unit of measurement is related to another unit of measurement. A unit conversion factor is a fraction in which **the numerator and denominator represent the same quantity, but in different units**. The ratio can be simplified to one. There is usually a **1** in the **numerator or denominator**.

The fraction below is a unit conversion factor that can be used to covert miles to feet. **Note that it can be simplified to one.**



Unit Analysis Method:

-First, make a unit cancellation table and write the number and units you are starting with in the left top box.

-Next, find the conversion factor for the units you want. When choosing a unit conversion factor, choose the one that cancels the units you **have** and replaces them with the units you **want**.

-Write unit conversion factor as a ratio in the form of a fraction (equal to 1, where numerator and denominator represent the same quantity, but in different units. Write it in such a way that all the units will cancel, except the unit you WANT. **When using unit analysis, the correct format will be such that after** mult/div **, the "have" ("from") units will cancel, and the answer will equal the "want" (""to") units.**

-Then, cancel units, and multiply/divide Write the final answer with the unit you "want" (the units you are converting "to"). top row: mult bottom row: divide

How many inches are there in 18 feet? Use unit analysis.

18 ~~feet~~	12 inches
	1 ~~foot~~

18 x 12 = 216. The answer is 18 feet = 216 inches.

How many inches are in 42 miles?

42 ~~mi~~	5280 ~~ft~~	12 in
	1 ~~mi~~	1 ~~ft~~

42 × 5280 × 12 in

42 miles = 2 661 120 inches

How many miles are in 158 400 inches?

158 400 ~~in~~	1 ~~foot~~	1 mi
	12 ~~in~~	5280 ~~ft~~

158 400 ÷ 12 ÷ 5280

158 400 2.5 miles

The important points are:

-Write the conversion as a fraction (that equals one)
-Multiply it out (leaving all units in the answer)
-Cancel any units that are both top and bottom

Go back and try p. 5 example 1 ii using unit conversion

Try it:

1. **Convert 2 mi. to in.** 2 miles = 126 720 inches

$$\frac{2\,\cancel{mi} \mid 5280\,\cancel{ft} \mid 12\,in}{\quad \mid 1\,\cancel{mi} \mid 1\,\cancel{ft}}$$

2 x 5280 x 12 in

2. **Convert 380160 in. to mi.** 80 160 inches = 6 miles

$$\frac{380160\,\cancel{in} \mid 1\,\cancel{ft} \mid 1\,mi}{\quad \mid 12\,\cancel{in} \mid 5280\,\cancel{ft}}$$

380160 ÷ 12 ÷ 5280

3. Anne is framing a picture. The perimeter of the framed picture is 136in.
   a) What will be the perimeter of the framed picture in feet and inches?

$$\frac{136\,\cancel{in} \mid 1\,ft}{\quad \mid 12\,\cancel{in}}$$

$\frac{136}{12} = 11.3\,ft$

$11 \times 12 = 132$

$136 - 132 = 4$

$\boxed{11\,ft\ 4\,in}$

   b) The framing material is sold by the foot. It costs \$1.89/ft. The material is not sold in partial feet. What will be the cost of material before taxes?

need 12 feet

(1.89)(12)

$\boxed{\$22.68}$

4. A school council has 6 yd of fabric that will be cut into strips 5 in. wide to make decorative banners for the school dance. How many banners can be made?

$$\frac{6\,\cancel{yd} \mid 3\,\cancel{ft} \mid 12\,in}{\quad \mid 1\,\cancel{yd} \mid 1\,\cancel{ft}}$$

6 x 3 x 12 in = 216 in

216 ÷ 5 = 43.2

Can make ~~259~~ 43 complete banners.

(not enough to make 44)

Practice

Hint
1 foot = 12 inches
1 yard = 3 feet
1 mile = 1760 yards

1. Express each measurement in inches.

9 ft 9+1=10 ft 10(12)

a) 6 ft 2 in. = 74 in. (6x12)

b) 4 ft 9 in. = 57 in. (4x12)

c) 2 yd = 72 in.
6 ft 6x12

d) 3 yd 1 ft = 120 in.

e) 5 yd 2 ft 3 in. = 207 in. (15 ft)
15+2 =17 ft 17x12 = 204"

f) $5\frac{1}{2}$ yd = 198 in.
(5.5)(3) = 16.5' (16.5)(12)

2. Pam is putting quarter-inch pieces of plywood in a stack. She has stacked 20 pieces so far. How high is the stack?

20 (0.25 in) = 5 inches.

3(5280)=15840 ft
15840 +9 +1
3(3)= 9 ft

3. Express each measurement in feet.

a) 7 yd 2 ft = 23 ft (21 ft)

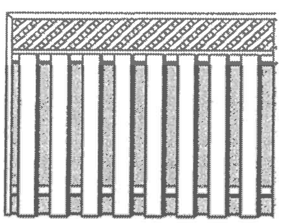
b) 12 yd 1 ft = 37 ft (36 ft)

c) 2 mi = 10560 ft
(5280)(2)

d) 3 mi 5 yd 1 ft = 15850 ft

e) $\frac{1}{2}$ mi = 2640 ft
(5280) ÷ 2

f) $2\frac{3}{4}$ mi = 14520 ft
(2.75)(5280)



4. Ray is building a fence using panels that are sold in 8 ft lengths. The perimeter of the area for the fence measures 32 yd. How many fence panels should he buy?

32(3) = 96 ft

96 ÷ 8 = 12   12 fence panels

5. Riley bought 50 feet of rope. He cut off pieces that total 34′ 8″ in length to use as tie-downs on his boat. How much rope does he have left?

$34'8'' = 34\frac{8}{16}' = 34\frac{1}{2}'$

$50 - 34\frac{1}{2} = 15\frac{1}{2}$ feet or 15' 8 inches

Textbook work: p. 11 #3, 4, 5, 8, 9, 11a, 12,13, 14, 18

SECTION: 1.3 RELATING SI AND IMPERIAL UNITS p. 16

The SI (Metric) system of measures is based on powers of ten.

Each measurement in the imperial system relates to a corresponding measurement in the SI system.

This table shows the approximate relationships between imperial units and SI units.

SI UNITS TO IMPERIAL UNITS	IMPERIAL UNITS TO SI UNITS
1 mm = $\frac{4}{100}$ in 0.25	1 in = 2.54 cm
1 cm = $\frac{4}{10}$ in 0.4	1 ft = 30.48 cm 1 ft = 0.3048 m
1 m = 39 in 1 m = 3 $\frac{1}{4}$ ft 3.25 1 m = 3.280839895 ft	1 yd = 90 cm 1 yd = 0.9 m
1 km = $\frac{6}{10}$ mi 0.6 1 km = 0.62137119 mi	1 mi = 1.609344 km

We can use the data in the table above and on front page of booklet to convert between SI units and imperial units of measure.

To convert between units, we can use our unit cancellation table, or can mult/divide.

Example 1:

A bowling lane is approximately 19 m long. What is this measurement to the nearest foot?

19m ≈ 62 feet

19m	3.280839895 ft
	1m

19 × 3.280839895 ft

(Think of a metre stick to know the length of a meter. Think of a long school ruler to think of the length of a foot (12 inches). There are about 3 feet in a metre. The answer in feet should be more than metres.)

smaller → larger $\boxed{\text{DIVIDE}}$

larger → smaller $\boxed{\text{MULTIPLY}}$

Example 2:

After meeting in Emerson, Manitoba, Hannah drove 62 mi south and Faith drove 98 km north. Who drove farther?

(Convert one to the same unit as the other. Choice. Both in miles or both in km.)

Convert 62 mi to km: 99.7793 km is more than 98 km.

proportion $\frac{mi}{km}$ $\frac{62}{x} = \frac{1}{1.609344}$ cross mult.

62 ~~mi~~	1.609344 km
	1 ~~mi~~

62 × 1.609344 km

Hannah drove farther.

Convert 98 km to mi: 60.8943 mi is less than 62 mi.

98 ~~km~~	0.6213719
	1 ~~km~~

98 × 0.6213719

Hannah drove further.

Example 3:

Alex is 6 ft 2 in tall. To list his height on his driver's license application, Alex needs to convert this measurement to centimeters.

a) What is Alex's height to the nearest centimeter?

6 ft 2 in
$6 \times 12 + 2$
74 in.

74 in $\times 2.54$
$= 187.96$
$= 188$ cm.

b) Use mental math and an estimation to justify that the answer is reasonable.

(Remember there are about 3 feet in a metre. And a metre is the length of a metre stick.)

6 feet is about 2 metres
$(6 \div 3)$
188 cm is 1.88 m

We may need to complete <u>more than one conversion</u> if what we want is not on the conversion factor chart.

☞For example, you may need to change from Imperial to SI first , then convert the SI units to the desired unit.

← divide

mult →

Remember:

×10 ×10 ×10 ×10 ×10 ×10

Kilometer	Hectometer	Decameter	Meter	Decimeter	Centimeter	Millimeter
1000 m	100 m	10 m	1 m	1/10 m	1/100 m	1/1000 m

÷10 ÷10 ÷10 ÷10 ÷10 ÷10

Each metric unit gets 10 times bigger. ←

Each metric unit gets 10 times smaller. →

Try it:

At least once a year, a truck will get stuck on the High Level Bridge in Edmonton. The bridge has a low clearance of 10' 6". A truck driver knows that her semitrailer is 3.3 m high. Will her vehicle fit under the bridge? Or will she be stopping traffic? Justify the answer.

3.3 m	3.280839895 ft	12 in
	1 m	1 ft

= 129 in

10 × 12 = 120
120 + 6 = 126
10'6" = 126"

Semi trailer 129"
bridge clearance 126"
No she will not fit. She will stop traffic

alternate method

126 in	1 ft	0.3048 m
	12 in	1 ft

126 ÷ 12 × 0.3048
= 3.2 m ← bridge
126 in = 3.2 m

bridge clearance 3.2 m
semi trailer 3.3 m

How could the truck driver get the trailer to fit?

Let air out of tires!

Metric Review:

Each of the following objects have been measured with inappropriate units. Convert them to more suitable units.

1. thickness of a dime 0.00122 m convert to mm
   1.22 mm multiply!
2. height of a basketball player 2100 mm convert to m
   2.1 m or 210 cm
3. driving distance from Pincher Creek to Taber is 14 900 000 cm convert to km
   149 km.

1. About how many centimetres are there in an inch?
1 in. ≐ 2.54 cm

2. About how many centimetres are there in a foot?
1 ft ≐ 30.48 cm   2.54x12

3. About how many centimetres are there in a yard?
1 yd ≐ 91.44 cm   30.48x3

4 Circle the greater measurement in each pair.

15.24 cm
a) 6 in. or 10 cm

1.25m
b) 5 ft or 125 cm
4.1 ft

30ft x 0.3048 = 9.144 m
c) 10 yd or 10 m

d) 25′ or 8 m
25(0.3048) 7.62

5 Circle the larger unit in each pair.

centimetre or metre
millimetre or decimetre
hectometre or kilometre
decametre or centimetre

6 How many centimetres are in 5 m?
1 m = 100 cm, so 5 m is 5 m × 100 cm/m = 500 cm

7 How many metres are in 8.5 km?
1 km = 1000 m, so 8.5 km × 1000 m/km = 8500 m

8 a) 1 cm = 10 mm
b) 1 m = 100 cm
c) 1 km = 1000 m
d) 45 cm = 0.45 m
e) 1330 m = 1.3 km
f) 45 mm = 4.5 cm

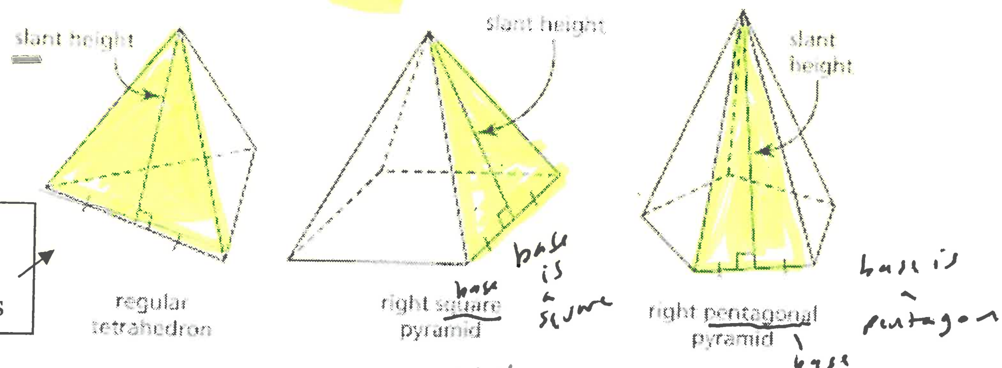
Textbook work: p. 22-23 #4-6, 7a, 8, 10, 15

Surface area is measured in units squared.

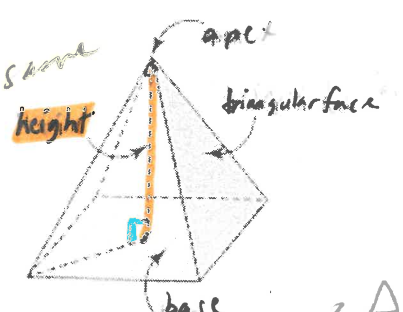
Right Pyramids

- The surface area is the total area on the surface of an object.
- A right pyramid is a 3 – dimensional shape that has triangular faces and a base that is a polygon. The apex of the shape is directly above the centre of the base.
- A right **regular** pyramid has a base that is a **regular polygon** (all sides equal), which makes all the lateral faces the same.
- The shape **of the polygon** determines the name **of the pyramid**.

Tetrahedron:
base and sides are **equilateral** triangles

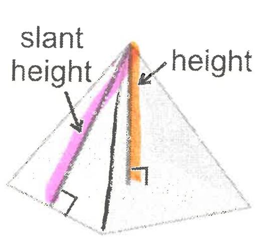


- The triangular faces meet at a point called the apex.
- The height of the pyramid is the perpendicular distance from the apex to the centre of the base.

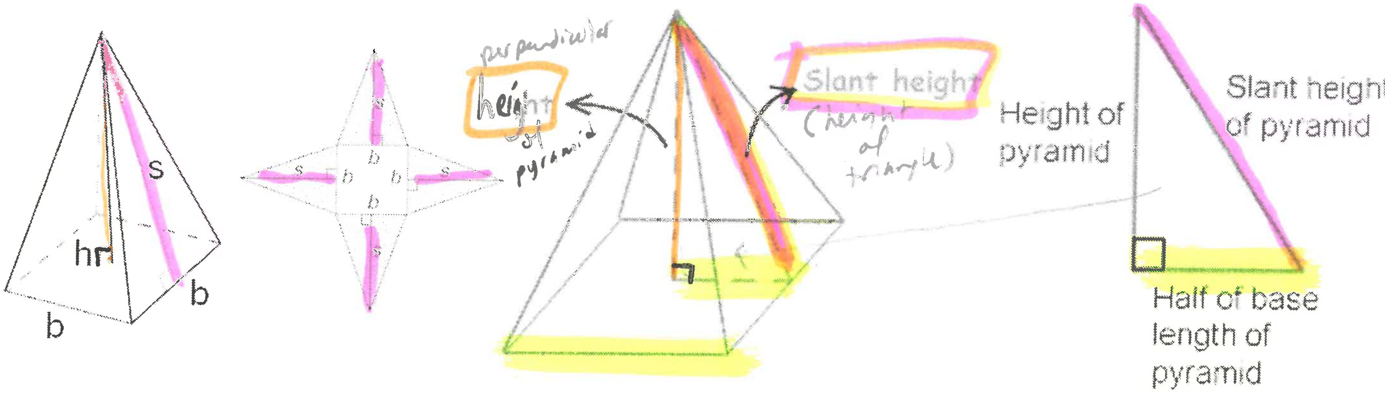


all sides the same

- When the base of a pyramid is a regular polygon, the triangular faces are **congruent** (all the same). All **regular** pyramids have a slant height, which is the height of a lateral face. The slant height of the regular right pyramid is the height of a triangular face (a lateral face).



- The slant height of a right pyramid is the hypotenuse of the right triangle formed by the height and half the base length. (***r*** is half the length of the base).

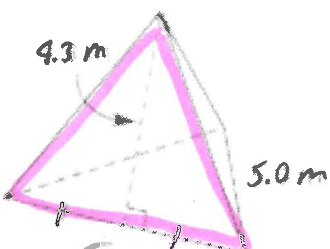


- The Surface area of a right pyramid is the sum of the areas of the triangular faces and the base.

square base 4 equal Δ
rectangle base 2 pair equal Δ

- **Example 1**

Calculate the surface area of this regular tetrahedron to the nearest square meter.



Find the area of one lateral face (one equilateral triangle). (area of triangle: $\frac{bh}{2}$)

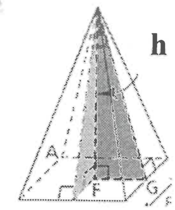
$$A = \frac{bh}{2} = \frac{(5.0)(4.3)}{2} = \frac{21.5}{2} = 10.75\,m^2$$

A tetrahedron is made up of 4 equilateral triangles. Take the area of one triangle and multiply by 4 to get the surface area of this tetrahedron.

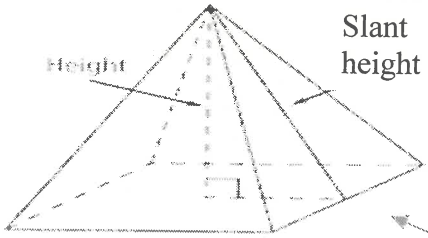
one triangle area = $10.75\,m^2$
tetrahedron surface area: $(10.75)(4)$
$= 43\,m^2$

Example 2

perpendicular pyramid

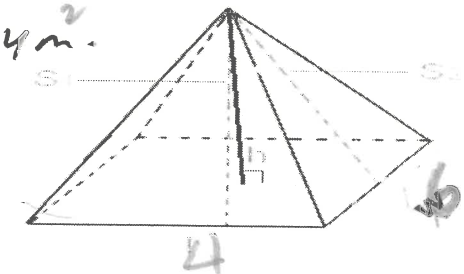


A **right rectangular pyramid** has base dimensions **4m by 6m, and its height is 8m.** Calculate the surface area of the pyramid to the nearest square metre.



"Right rectangular pyramid" means that the base is a rectangle. It has 4 lateral faces that are 2 pairs of Congruent (equal) triangles.

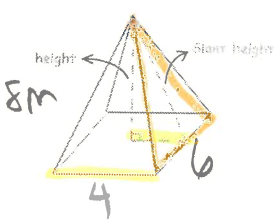
- We can find the rectangle $\boxed{\textbf{base}}$: $A = LW = (4)(6) = 24\,m^2$.

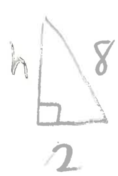


- For the area of the **triangles**, we need the **base** (*which we know*) and the **height** (which we need). The height of the triangle is *the slant height of the pyramid.* <u>The slant height is actually the hypotenuse of the triangle that is formed by the **height** and **half of the base length** of the pyramid</u> .

For the left and right side triangles, visualize the right triangle to find the slant height (height of those triangles).

height = 8m r = $\frac{1}{2}$ length of base $\frac{1}{2}(4) = 2$. Find the slant height (hypotenuse)





$h^2 = 2^2 + 8^2$

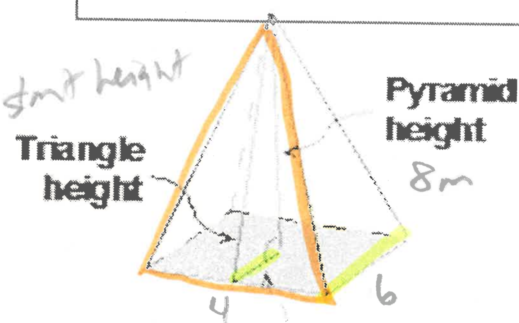
$\sqrt{h^2} = \sqrt{68}$

$h = 8.2462$ ← slant height of pyramid (height of side triangle)

$A = \frac{bh}{2} = \frac{(6)(8.2462)}{2} = 24.7386$ side Δ

Now find the **area of one side triangle.**

Repeat the process for the triangles at the front and back.



$\frac{1}{2}$ of 6 $= 3$

$h^2 = 8^2 + 3^2$

$h^2 = 64 + 9$

$\sqrt{h^2} = \sqrt{73}$

$h = 8.5440$

↑ slant height of front of pyramid height of front Δ

$A = \frac{bh}{2}$

$= \frac{(4)(8.5440)}{2}$

front Δ $A = 17.0880$

Total area of this pyramid = 2 triangles + two triangles + base

$SA = 2(24.7386) + 2(17.0880) + 24$

$= 107.6532$

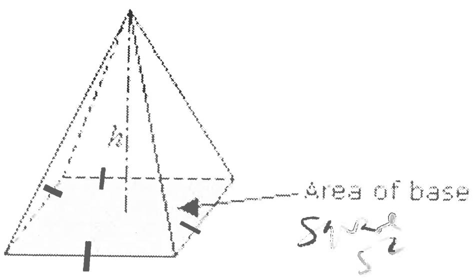
$SA = 108\,m^2$

Formula: Surface area of Right pyramid with Regular polygon base

Instead of finding the sum of all the faces of a right regular pyramid, we can use this formula. (**base** has to have **all sides equal** for this formula). "**s**" is the slant height.

Surface area = $\frac{1}{2}$ s (perimeter of base) + base area

$\underbrace{\tfrac{1}{2} s \text{ (perimeter of base)}}_{\text{lateral area } \triangle}$ $\underbrace{\text{base area}}_{\text{area of base}}$



Square
s^2

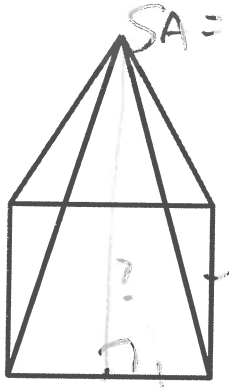
not including square base

base is a square

Example 3: The surface area of the lateral triangular faces on a right squared pyramid is 3000in^2. The side length of its base is 30 in. Determine the ~~slant~~ height of the pyramid. to tenth of an inch.

If the base is a **regular** polygon with all sides equal, then **all the triangles** will be congruent. (equal) (the same)

SA = 3000 in^2



$SA = \frac{1}{2} S$ (perimeter of base) + ~~base area~~

$3000 = \frac{1}{2} S \left[4(30)\right]$

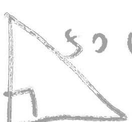
$3000 = \frac{1}{2} S (120)$

$\frac{3000}{60} = \frac{60S}{60}$

$50 = S$

slant height = 50 in.

not squared



$\frac{1}{2}$ of 30

$h^2 + 15^2 = 50^2$

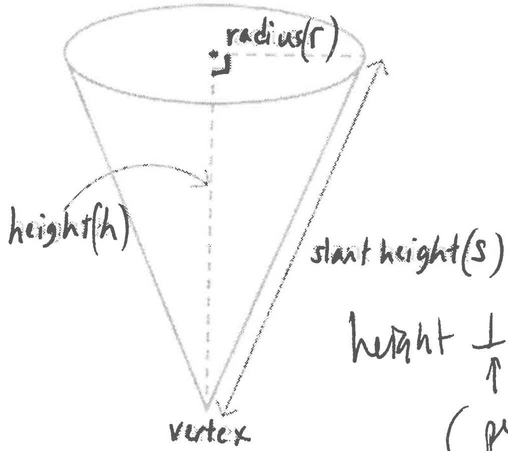
$h^2 + 225 = 2500$
$-225 \quad -225$

$\sqrt{h^2} = \sqrt{2275}$

$h = 47.7$ in

Height of pyramid 47.7"

Right Cones



height $\perp$ radius
↑
(perpendicular)

Surface area formula:

area of circle	+	lateral area

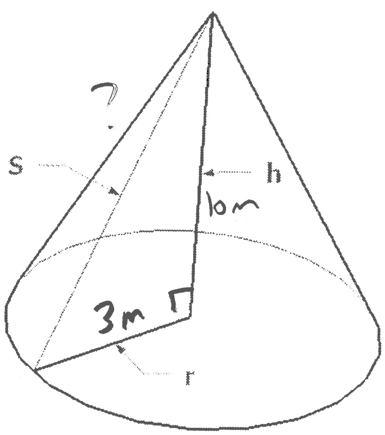
$SA = \pi r^2 + \pi rs$

r = radius

s = slant height

Example 1

A right cone has a base radius of 3 m and a height of 10 m. Calculate the surface area of this cone to the nearest square metre.



$SA = \pi r^2 + \pi rs$

To find the surface area, we need the slant height.

The **radius** and height of a cone are **perpendicular**. The slant height forms the hypotenuse of the right triangle that contains the height and radius.

Use Pythagoras.

$s^2 = 3^2 + 10^2$

$s^2 = 9 + 100$

$\sqrt{s^2} = \sqrt{109}$

$s = 10.4403$

$r = 3$

$s = 10.4403$

$SA = \pi r^2 + \pi rs$

$= \pi(3)^2 + \pi(3)(10.4403)$

$= \pi(9) + \pi(31.3209)$

$= 9\pi + 31.3209\pi$

$= 40.3209\pi$ multiply

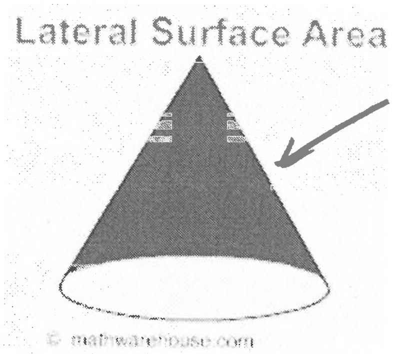
$SA \approx 127\ m^2$

Example 2

not including circle base

r = 5 cm

The lateral surface area of a cone is 220 cm^2. The diameter of the cone is 10 cm. Determine the **height of the cone** to the nearest tenth of a centimetre.



220 cm^2

circle

$SA = \cancel{\pi r^2} + \pi r s$

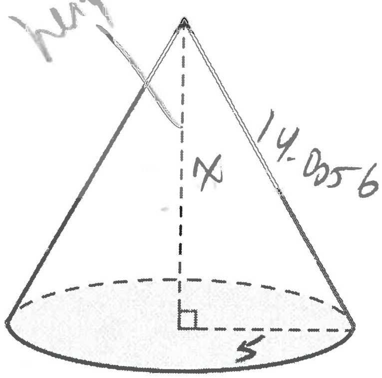
$SA = \pi r s$

$220 = \pi(5)(s)$

$\dfrac{220}{5\pi} = \dfrac{(5\pi)s}{5\pi}$

$14.0056\text{ cm} = s$

$220 \div 5 \div \pi$



$5^2 + x^2 = 14.0056^2$

units not squared

$x = 13.1\text{ cm}$
height

Do all at once in calc.

$\sqrt{(14.0056)^2 - 5^2}$

Text work: pyramids p. 34 #4,5, 8a, 13 cones #6,7,8b,11

1.5 VOLUME OF RIGHT PYRAMIDS AND RIGHT CONES (p. 36)

Write in COMPLETE SENTENCES.
Use your textbook – pages 36 to 41.

1. What is volume? (*yellow box p. 36*)

   Volume is the amount of space an object occupies.

   It is measured in cubic units.. (units^3)

2. What is **capacity**?

   Capacity is the amount of material a container holds.

It is measured in cubic units or capacity units (like litre, milliliter, ounce, gallon, etc.)

3. Write at least one or two examples of the units we use for volume.

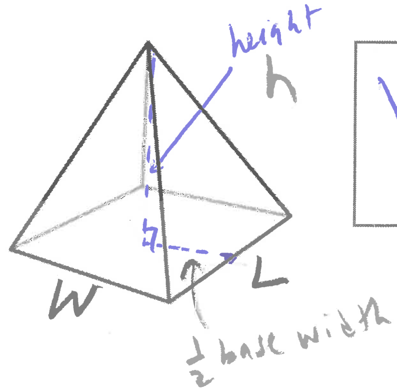
   cm^3

   yd^3

   m^3

   ft^3

4. Write the **formula** for the volume of a **right rectangular pyramid**. What does each of the letters mean? Label this diagram (use a ruler) like on p. 39.



$$\boxed{V = \frac{1}{3} lwh \quad \frac{1}{3} LWH}$$

l - length

w - width

h - height (perpendicular height of pyramid)

5. What theorem might will you use to find missing parts of the formula? (p. 38)

Pythagoras.

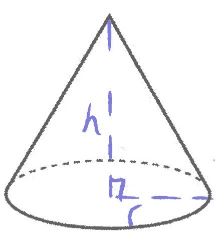
6. Read the examples on pages 38 and 39. Do you understand them? If not, read them again. Identify parts that confuse you.

p.38 for volume we need lwh.
If given slant height, we use Pythagoras.
We use $\frac{1}{2}$ side length of base of pyramid for base of Δ.
$$\text{slant height}^2 = (\tfrac{1}{2}\text{ side length})^2 + (\text{height of pyramid})^2$$

7. Try the "Check Your Understanding" question on page 39. Check your answer.

$$V = \frac{1}{3} LWH$$
$$V = \frac{1}{3}(4.7)(3.6)(6.9)$$
$$V \approx 38.9\,m^3$$

8. Write the formula for the volume of a right cone. What does each of the letters mean? Label this diagram (use a ruler) like on p. 40.



r - base radius

h - height (⊥ height of cone)

$$V = \frac{1}{3}\pi r^2 h$$

9. Read the examples on pages 40 and 41. Do you understand them? If not, read the examples again. Identify parts that confuse you.

diameter = 12 in

radius = $\frac{1}{2}(12) = 6$ in.

10. Try the "Check Your Understanding" question on page 40. Check your answer.

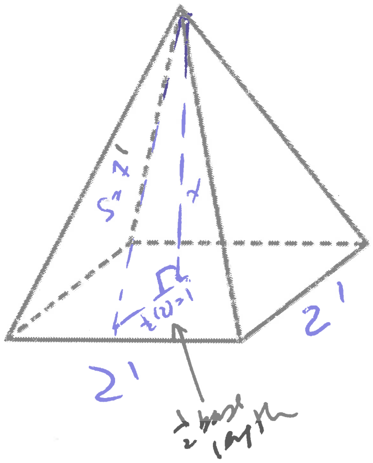
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (4)^2 (13)$

$V \approx 218 mm^3$

Try it:

Try the "Check Your Understanding" question on page 38 Check your answer.



hyp

Slant height = 7 ft

Square pyramid all sides = 2

$x =$ height

$\frac{1}{2}$ base length $= \frac{1}{2}(2) = 1$

$7^2 = 1^2 + x^2$

$49 = 1 + x^2$

$-1 \quad -1$

$\sqrt{48} = \sqrt{x^2}$

$\sqrt{48} = x$

$6.9282 = x$

height

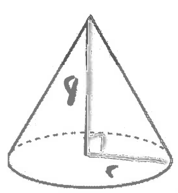
← need

$V = \frac{1}{3} lwH$

$V = \frac{1}{3}(2)(2)(\sqrt{48})$

$V \approx 9 ft^3$

Try the "Check Your Understanding" question on page 41 Check your answer.



$V = 300 m^3$

$V = \frac{1}{3}\pi r^2 h$

$3(300) = (\frac{1}{3}\pi r^2 (8))3$

$\frac{900}{8\pi} = \frac{\pi r^2 (8)}{8\pi}$

$\sqrt{900 \div 8 \div \pi}$

$\sqrt{\frac{900}{8\pi}} = \sqrt{r^2}$

$6 \approx r$

radius $\approx$ 6 m.

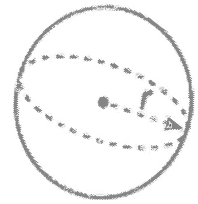
Textbook PRACTICE: Choose at least 7 of the following questions to try, starting on page 42: 4 to 13. Check your answers with the answers in the back of the book.

1.6 VOLUME and SURFACE AREA of SPHERES (p. 47)

- SURFACE AREA OF A SPHERE p. 45

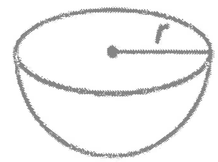
To find the surface area of a sphere, use this formula:

$SA = 4\pi r^2$

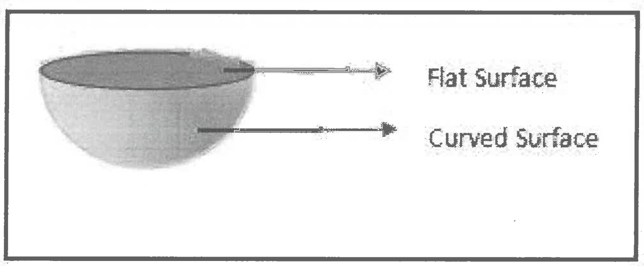


To find the surface area of a hemisphere, use this formula:

$SA = 3\pi r^2$

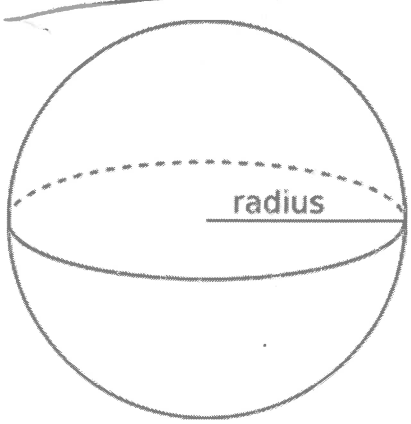


(Hemisphere is half of a **sphere**. It is half the surface area of curved surface of the sphere, PLUS the circle that is the flat surface of the hemisphere. $\therefore SA = \frac{1}{2}(4\pi r^2) + \pi r^2 = 2\pi r^2 + \pi r^2 = 3\pi r^2$)



Example 1

A glass sphere has radius 25 cm. What is the surface area of the sphere, to the nearest square centimetre?



$SA = 4\pi r^2$

$= 4\pi(25)^2$

$SA = 7854\ cm^2$

Example 2

A globe has surface area 2735 cm². Find the radius of the globe, to the nearest tenth of a centimetre.

$SA = 4\pi r^2$

$2735 = 4\pi r^2$

$\frac{2735}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\sqrt{\frac{2735}{4\pi}} = r$

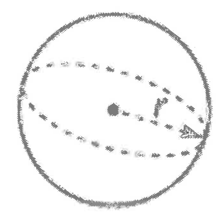
$\sqrt{(2735 \div 4 \div \pi)} =$

$14.8 \text{ cm} = r$

- **VOLUME OF A SPHERE p. 49**

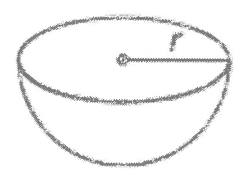
To find the volume of a sphere, use this formula:

$V = \frac{4}{3}\pi r^3$



To find the volume of a hemisphere, use this formula:

$V = \frac{2}{3}\pi r^3$



(For volume of hemisphere, we simply divide the volume of sphere in half.)

Example 3

4 radius

A sphere has diameter 8 yd. What is the **volume of the sphere,** to the nearest cubic yard?

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (4)^3$$

$$V = 268 \text{ yd}^3$$

$4 \times \pi \times 4^3 = \div 3 =$

Example 4

A hemisphere has radius 6.0 cm.

a) What is the **surface area** of the hemisphere to the nearest tenth of a square centimetre?

$$SA = 3\pi r^2$$

$$SA = 3\pi (6)^2$$

$$SA = 339{,}2 \text{ cm}^2$$

b) What is the **volume** of the hemisphere to the nearest tenth of a cubic centimetre?

$$V = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi (6)^3$$

$$V = 452{,}4 \text{ cm}^3$$

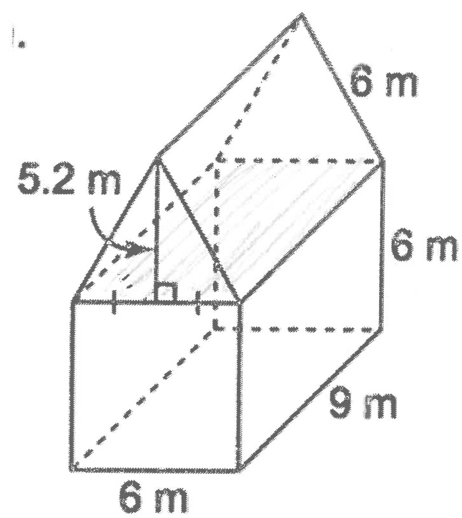
Textbook work: p. 51 #3c, 4c, 5, 8, 9, 10, 11

1.7 SOLVING PROBLEMS INVOLVING OBJECTS p. 55

- A composite object compromises of two or more distinct objects.
- To determine the **volume of a composite object**, identify the distinct objects, calculate the volume of each object and then add the volumes.
- To calculate the **surface area of a composite object**, the first step is to determine the faces that make up the surface area. Then Calculate the sum of the areas of those faces.
- Surface Area of a Composite Object

Be careful. Look at the object to the right. To calculate the surface area, it's not just calculating the sum of areas of a triangular and rectangular prism. Note that these two prisms are attached. **We don't include the base of the triangular prism or the top of the rectangular prism.** If you like, you add the areas of the two shapes and then subtract the **two** rectangles that **overlap** that are not part of the exterior surface area.

minus $2(6)(9)$



Example 1

A sphere of flavoured ice is served in a cylinder shaped paper cup. The cup has a diameter 6 cm and height 10 cm. For the moment before the ice starts melting, the sphere has the same diameter as the cup. To the nearest cubic centimetre, how much space is left inside the cup? (Hint: One-half of the sphere is below the rim of the cup.) Sketch it!

$V_{cylinder} = \pi r^2 h$
$= \pi(3)^2(10)$
$= 90\pi$

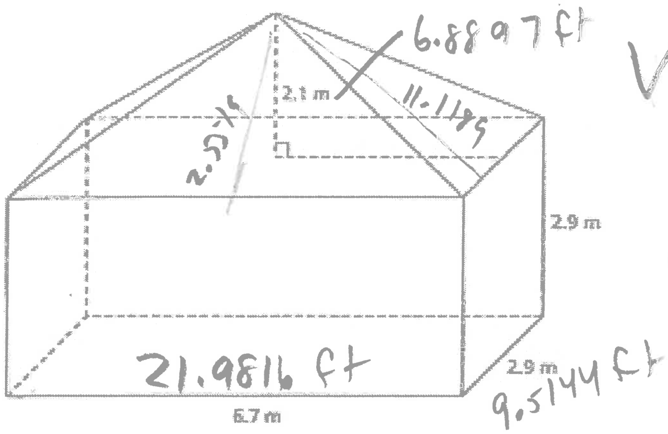
$V_{hemisphere} = \frac{2}{3}\pi r^3$
$= \frac{2}{3}\pi(3)^3$
$= \frac{2}{3}\pi(27)\,9$
$= 18\pi$

Space inside
$=$ cyl − hem
$= 90\pi - 18\pi$
$= 72\pi$
$= 226\,cm^3$

Example 2

a) Determine the volume of this object to the nearest tenth of a cubic meter. 69.9 cm³

(You are finding out **how much space is INSIDE** this object.)



$V_{rect\,prism} = LWH$

$= (6.7)(2.9)(2.9)$

$= 56.347$

$V_{pyra} = \frac{1}{3}LWH$

$= \frac{1}{3}(6.7)(2.9)(2.1$

$= 7.1766$

Volume composite object: prism + pyra

$= 63.5\ m^3$

SKIP!

b) Calculate the surface area of this object to the nearest square foot. (Hint: convert all dimensions BEFORE calculating the surface area). (You are calculating the **sum** of **areas** of **all the flat surfaces (faces)** on **OUTSIDE** of this shape.)

$SA_{prism} = 2(lw + wh + lh)$

$= 2[(21.9816)(9.5144) + (9.5144)(9.5144) + (9.5144)(21.9816)]$

$= 2(209.1417 + 90.5238 + 209.1417)$

$= 1017.6145$

$SA\ pyramid = 2\left(\frac{(21.9816)(2.5519)}{2}\right) + 2\left(\frac{(2.9)(11.1189)}{2}\right)$

without base or top ← 2 rectangles overlap

$= 56.0946 + 32.2448$

$= 88.3396$

need slant height

$2.1^2 + 10.9908^2 = x^2$

$x = 11.1189$

side Δs

front Δ

$2.1^2 + 1.45^2 = y^2$

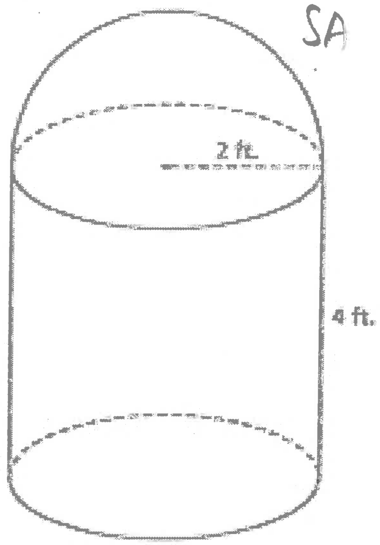
$2.5519 = y$

Composite: prism + pyr (without 2 rectangles)

$1106\ ft^2$

Example 3

Calculate the surface area of this object to the nearest square foot. (88 ft²)



SA Cylinder without 2 circles (overlap)

$2\pi r h$

$2\pi(2)(4)$

16π

SA hemisphere including circle

$3\pi r^2$

$3\pi(2)^2$

12π

composite

$16\pi + 12\pi$

28π

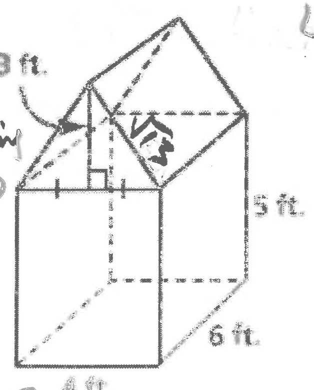
88 ft²

Example 4

A tool shed is formed by a rectangular prism with a triangular prism as its roof. Determine the surface area of the tool shed to the nearest square foot.

You can calculate the surface area without knowing the formulas for the two shapes that make up the composite. Just look at what rectangles and triangles make up exterior of the tool shed. ***Note that for the two equal rectangles of the roof, you can use Pythagoras to find the width.*** (see p. 58 for similar example). 155 ft²

4 walls + roof



overlapping 2 rect.

not including base

4 walls: $2(6\cdot5) + 2(4\cdot5)$

$= 60 + 40$

$= 100 \text{ ft}^2$

roof: Pyth to find width of roof rectangles

3, h, 2 ($\tfrac{1}{2}$ of 4)

$3^2 + 2^2 = h^2$

$9 + 4 = h^2$

$h = \sqrt{13}$

2 rectangles: $2(\sqrt{13})(6) = 12\sqrt{13}$

$= 43.2666$

2 triangles: $2\left(\frac{bh}{2}\right) = 2\left(\frac{4\cdot3}{2}\right) = 12$

Textbook work: p. 59 3a, 3b, 3d, 4a, 4d, 6

Shed area: $100 + 43.2666 + 12$

$\approx 155 \text{ ft}^2$